STAT 696, Spring 2011
Homework 6 Problems
due Thursday April 7 (after Spring Break)

2 Problems. Please follow the Lab report directions off the homework web page.

1. What is the effect of the variogram on kriging?

(a) In class, we showed that the kriging weights are unaffected by a change in the value of the sill from c_1 to c_1^*. Using the matrix formulas for the kriging variance and generalized least squares estimate of the global mean, show that

$$\sigma_K^{2*} = (c_1^*/c_1)\sigma_K^2$$

which is (c_1^*/c_1) times as large as the kriging variance when the sill is c_1, and

$$\hat{m}_{GLS}^* = \hat{m}_{GLS}$$

so that changing the sill has no effect on the GLS estimate of the global mean.

(b) Let the Measurement Error model for the nugget effect be:

$$c_0 = c_{MS} + c_{ME}$$

Show that the kriging variance at $s_i = s_0$ is

$$\text{Var}(\hat{Z}_0 - Z_0) = \boldsymbol{w_1'\Gamma_0^*} - \lambda + c_{ME}$$

Problem 2 on back.

2. Compare ordinary kriging (OK) with universal kriging (UK) for the **Soil pH data**. Data on soil pH comes from an observational study in which samples of soil were collected at the nodes of a grid with 11 rows $\times$ 11 columns. The pH of the samples were measured in a laboratory.

The data is available off the class web page:

`http://www.rohan.sdsu.edu/~babailey/stat696/soilph.dat`

Use the R `read.table` command with the `header=T` option. (You do not need to make your own labels!)

(a) OK: Fit a 2nd order polynomial trend to the data. Let $m(x, y) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 xy + \beta_5 y^2$. Use the `lm` function. Now, using the residuals, fit a variogram model (exponential), and use the `krig` function to obtain kriging predictions and variances for the data. You can use the kriging variances, but do not forget to add back on the estimated trend for the final predictions!.

(b) UK: Repeat (a) using Universal Kriging. Use the formula statement for the variogram and kriging with the second order polynomial.

(c) Make plot of OK vs UK predictions and OK vs UK variances. How do the methods compare?

(d) Obtain an OK and UK kriging predictions and variances at a new location $\boldsymbol{s}_0$ close to middle of the grid. You can use the `identify` function to identify the coordinates of the point. How do the methods compare?