STAT 700
Homework 1 Problems
due Wed. Sept. 7

2 Problems. Show all work.
This is an individual assignment and should be **handwritten**.

1. Let $X_1, X_2, \ldots, X_n$ be independent random variables from the $N(\theta_1, \theta_2)$ distribution where $\{(\theta_1, \theta_2) : -\infty < \theta_1 < \infty, 0 < \theta_2 < \infty\}$

Let's find the MLEs by the following steps:

(a) Write down the likelihood function and call it $L(\theta_1, \theta_2)$.

(b) Write down the log-likelihood function and call it $\log L(\theta_1, \theta_2)$.

(c) Write down the two equations for the partial derivatives of the log-likelihood function with respect to θ_1 and θ_2.

(d) Find the maxiumum likelihood estimators for θ_1 and θ_2 and call them $\widehat{\theta}_1$ and $\widehat{\theta}_2$.

2. Let $X_1, X_2, \ldots, X_n$ be independent normal random variables with means μ_i and variances σ_i^2. Let $Y = \sum_{i=1}^{n} \alpha_i X_i$ where α_i are constants. Use moment generating functions to show that Y is normally distributed and find its mean and variance.

Recall, the moment generating function (mgf) of a normal random variable X with mean μ and variance σ^2 is
$$M_X(t) = e^{\mu t + \sigma^2 t^2 / 2}.$$