STAT 700
Homework 8 Problems
due Wed. Nov. 9

2 Problems. Please follow the Lab report directions off the homework web page and work in HW Groups.

1. This problem will fit growth curves to data for the change in an orthodontic measurement over time for several young subjects. Investigators at the University of North Carolina Dental School followed the growth of 27 children (16 males, 11 females) from age 8 until age 14. Every two years (age 8, 10, 12, 14) they measured the distance between the pituitary and the pterygomaxillary fissure, two points that are easily identified on x-ray exposures of the side of the head. It appears that there are qualitative differences between boys and girls, so we will just model the data from the male subjects. We will use the R dataset Orthodont (in the `nlme` library).

Follow the class example and make a dataset that consists of only the males. Call it `OthoM`.

(a) Fit two mixed models, `fit1` and `fit2`, one with a random effect for the intercept and one with random effects for both the intercept and slope. Provide the summary for each model.

(b) First test if the random intercept is needed in the model `fit1` and
test $H_0 : \sigma^2_{intercept} = 0$ against $H_1 : \sigma^2_{intercept} \neq 0$.

(c) Use the two models `fit1` and `fit2` test $H_0 : \sigma^2_{slope} = 0$ against $H_1 : \sigma^2_{slope} \neq 0$.
State your conclusion and state which is the "best" model.

(d) Using your answer to part (c). Give the estimates for all parameters in the "best" model. Give corresponding 95% CI for all the parameters using the R `intervals` function.

(e) How well does the "best" model fit the data? Include and examine the diagnostics plot of the residuals. You should also look at a Q-Q plot of the residuals.

(f) Use the R `augPred` function to plot the fitted values for the "best" model.

(g) The dataset `OthoM` is an example of `groupedData` and there is a formula associated with this type of data. Use the R function `formula(OrthoM)` to see the formula. There is also a `lmList` function that will fit a linear model to each Subject separately (see the help file). Obtain a fitted object using `lmList(OrthoM)`. Use the `plot` and `intervals` functions to plot the 95% CI of intercept and slope for each for subject. Examine the plot. Does your "best" model seem reasonable? (i.e., Does the plot indicate that there is a clear indication that one or both random effects is needed to account for the subject-to-subject variability?)

2. Let observations $Y_1, \ldots, Y_n$ be described by the relationship,

$$Y_i = \theta X_i^2 + \varepsilon_i$$

where $X_1, \ldots, X_n$ are fixed constants and the $\varepsilon_i, \ldots, \varepsilon_n$ are iid $N(0, \sigma^2)$.

Hint: For finding MLEs it might be useful to substitute another variable for X_i^2 and then replace it at the end.

(a) Find the MLE of θ (call it $\hat{\theta}$) and the MLE of σ^2 (call it $\hat{\sigma}^2$) following the methodology from Homework 1 Problem 1.

(b) Now, we will repeat (a) using matrix notation and follow the methodology from the Methodology for Linear Models in Matrix Notation in the Course Documents Folder in Lecture Materials After Midterm Exam Folder and the following the steps (i)-(iv), below. Note: $T = n$ in this problem.

(i) Write down the likelihood function following the form given in equation (5).

(ii) Write down the log-likelihood function following the form given in equation (6).

(iii) Write down the two equations for the partial derivatives of the log-likelihood function with respect to θ and σ^2 following the form given in equation (7) and (8).

(iv) Find the maxiumum likelihood estimators for θ and σ^2 following the form given in equation (12) and (13). (You should convince yourself that your answers to (a) are exactly the same as your anwers to (iv)!)

(v) Using matrix notation, find $E(\hat{\theta})$ and $Var(\hat{\theta})$. Note: This is not in the Methodology for Linear Models in Matrix Notation.

Not required and just for fun for students that have taken STAT 670AB, find the best unbiased estimator of θ. (Hint: CRLB).