

## Nonlinear Mixed-Effects Models: Growth of Soybean Plants

The **Soybean** dataset is described as follows: “The average leaf weight (in grams) of six plants chosen at random from each plot was measured at approximately weekly intervals, between two and eleven weeks after planting. The experiment was carried out over three different planting years: 1988, 1989, 1990. Eight plots were planted with each genotype in each planting year, giving a total of forty-eight plots in the study.” (ref. Pinheiro and Bates (2004, p. 287))

The nonlinear model for average leaf weight per plant  $Y_{ij}$  in plot  $i$  at  $t_{ij}$  days after planting is,

$$Y_{ij} = \frac{1 + \theta_{1i}}{1 + \exp(-(t_{ij} - \theta_{2i})/\theta_{3i})} + \varepsilon_{ij}$$

where

$$\boldsymbol{\theta}_i = \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \\ \theta_{3i} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} = \boldsymbol{\beta} + \mathbf{u}_i$$

with  $\mathbf{u}_i \sim N(\mathbf{0}, \mathbf{D})$  and  $\varepsilon_{ij} \sim N(0, \sigma^2)$ .